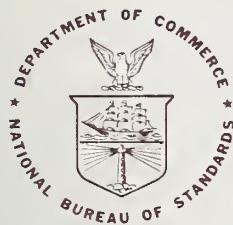




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**Beam-Profile Measurement of
Laser Pulses Using a Spatial
Filter to Sample the Hermite
Modes for a String of Pulses**

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Beam-Profile Measurement of Laser Pulses Using a Spatial Filter to Sample the Hermite Modes for a String of Pulses

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Beam-Profile Measurement of Laser Pulses Using A Spatial Filter to Sample the Hermite Modes for a String of Pulses

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As a first step in the development of a beam-profile measuring instrument for laser sources that is capable of determining the distribution of low-order (less than 25) Hermitian modes in a series of laser pulses, I designed and evaluated the three key parts of such an instrument. First, there is the telescope system which allows the incident laser beam to be phase, beamwidth, and beam center matched to the optical spatial filter. Second, there is a brief error analysis of the structure of the mismatch function between the beam out of the telescope and that expected by the filter. Finally, there is the detailed analysis and design of the computer-generated spatial filter that will cause the incident-laser beam to be cross correlated with the low-order Hermite modes and will create an array of light spots in the detector (Fourier transform) plane each of which can be uniquely related to a particular Hermite mode of the original laser pulse.

The principal conclusion is that the Hermite mode analysis can be done with better than 99 percent separation between modes, provided the phase between modes is uncorrelated from pulse to pulse when the filter has been fabricated with a two-level, gray-scale structure which samples the profile with either 0 percent, or 100 percent transmission.

Key words: computer simulation; laser beam profile; mode-matching analysis; spatial filter, target designators.

1. Introduction

1.1 Background Discussion

Beam-profile measurements of each laser pulse can increase the understanding of how a laser source operates. These measurements can allow eventual real-time control of the source and/or a complete evaluation of the source. There are two basic techniques that appear to be accurate for beam-profile measuring and that can be designed to allow moderately trained individuals to perform the necessary alignments and sequencing of the measurements: the focal plane [1] and the optical-filter techniques (ref. 2 and this paper). Each has its advantages and disadvantages. The appropriate one to use depends on the character of the laser source, the conditions where the measurements will be performed, and the desired results needed for decisions and/or control.

The focal-plane technique can use a single planoconvex lens around 2 m focal length to create a fluence pattern (J/cm^2) in the focal plane for the far field of the laser beam. This pattern is recorded with a TV camera system of appropriate spatial resolution and responsivity. The recorded energy density pattern can then be serially transferred to a minicomputer to determine a summary, such as the center of gravity of the beam, the moments of dispersion of the energy density, and the presence of far-field hotspots due to the laser source and optics. The prime advantages of this technique are: (1) the small number of optical elements and mechanical parts, (2) the non-predisposition to the characteristics of

the fluence distribution, and (3) the flexibility of using software for summarizing the results to determine whether or not the laser source is behaving as desired. The main disadvantages of this system are: (1) the seconds or more it takes to assess the desired information allow little chance for real-time assessment of individual laser pulses, and (2) there can be a phase-front curvature that changes the far-field pattern. The latter effect can be assessed by varying the position of the detector plane. This adjustment requires stability in the laser source. In brief, the focal-plane technique is best if the beam profile has temporal detail of no interest or is too variable to allow some optical processing technique to eliminate the unimportant information. The computer software permits a statistical reduction of the temporal variations.

The optical-filter technique is complimentary to the focal-plane technique. It works as follows: The pulse from a laser source goes through an optical filter which causes a complex spatial phase and amplitude modulation to be superimposed on the original laser pulse. This pattern is then intersected by a lens of one meter focal length whose function is to form the Fourier transform of the modified beam pattern. At selected transverse locations in the focal plane of the lens, there are spots of beam intensity which correspond to strength in key features of the original beam pattern. One example of key features is the electric field strength of the Hermite modes in the original beam. These spots of intensities are recorded by an array of detectors whose number can be less than 25.

The prime advantages of the optical filter technique are: (1) the filter can eliminate the unnecessary information about the beam profile and structure the desired information into a form that reflects the primary energy content of a laser pulse. This optical processing happens at the 10 ns rate, therefore the necessary processing is fast enough to allow a pulse-by-pulse assessment of the beam profile from a Q-switched 1.06 μm laser source. (2) The process is sensitive to the amplitude distribution of the laser beam rather than to the energy distribution. This means that the analysis is linear in the electromagnetic field and can thus be made so that we can have a sensitive measurement of the actual electromagnetic-field details instead of the energy-density details in a laser pulse. Knowledge of these details can permit diagnostics of the original laser source, such as the temporal stress changes of the laser rods and the temporal changes in the excitation processes. The prime disadvantages are: (1) advanced knowledge of the appropriate spatial filter is necessary so that the optical processing can be optimized. This advanced knowledge requires a basic structural stability of the laser source system in order for the reduction of extraneous information to be effective. (2) Because the optical filter technique is sensitive to the beamwidth, to the curvature of the phase front, and beam center, it may prove necessary in some measurement systems to include a lens system to match the laser source to the optical filter. This can imply a significant change in the size and complexity of the measuring apparatus for that system. This paper addresses such situations and describes in section 5 how they can be avoided.

1.2 Organization of This Paper

The aim of this paper is to evaluate by computer simulation a beam-profile measuring apparatus that is made up of three basic components, namely, the telescope unit, the optical

filter unit, and the detectors. Section 2 describes a telescope that can be used to cause the laser source, a target designator, to have a plane-wave front at the optical filter a beamwidth of 10 mm and a match of beam axis. Section 3 gives the mathematical formulas for the mismatch between the telescope and the assumed mode structure of the filter. Section 4 discusses the analysis of the computer-generated optical filter with detectors and the results of the computer simulation on how well a physical filter will perform the desired mode analysis. Finally, section 5 evaluates the concepts involved and, as a consequence of the computer simulations, makes conclusions in the context of these concepts. The appendices contain the computer programs used in simulation and some other analysis related to construction of modes and what response the detectors will record. The design is directed toward 1.06 μm wavelength. If other wavelengths are used then some of the design parameters will need to be adjusted.

2. The Telescope for Beamwidth and Phase-Front Control of a Laser Pulse

Here we assume four lenses are necessary to get complete control of the beamwidth and the phase-front curvature of a laser beam. This telescope is necessary only if the phase front of the laser source is not a plane wave and if the beamwidth, w_{10} , is not 10 mm at the spatial filter. Figure 2.1 shows the components of the telescope. The f_n are the focal lengths of the four simple lenses which are chosen to be $f_2 = 2000 \text{ mm}$, $f_4 = -f_2$, $f_8 = 2000 \text{ mm}$, and $f_6 = -f_8$. The distances d_n are the spaces between the lenses, the laser source, and the filter plane. We assume all lenses have an aperture of 120 mm. The q_n represent the complex beam parameters for an ideal Gaussian beam [3]. Thus, we have $q_n = i b_n + z_n$, where $b_n = w_n^2(\pi/\lambda)$, i is $(-1)^{1/2}$, w_n is the beamwidth at the n location, and $-z_n$ is the distance to a point where the Gaussian beam has the width w_n . We can use two relationships for the propagation of the laser beam.

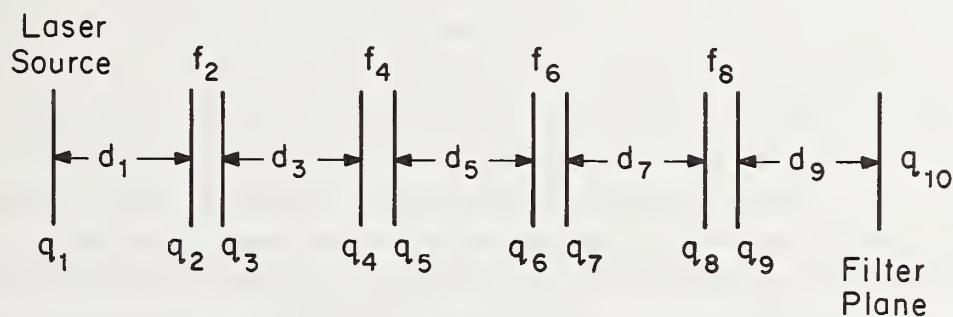


Figure 2.1. Beam adjustment diagram. Note that $d_{2n} = 0$ for the thin lenses and $f_{2n+1} = 0$ for the distances between the four lenses.

$$q_{n+1} = q_n + d_n \quad (\text{Type 1--a simple displacement.})$$

$f_n=0$ for this case.)

$$q_{n+1} = -f_n + \frac{f_n^2}{f_n - q_n} \quad (\text{Type 2--the thin lens law.})$$

$d_n=0$ for this case.)

To make efficient computation, we do the following: (Appendix A has the actual computer listing.) For processes from left to right of figure 2.1, we use

$$q_n = i \underline{b}_n + z_n$$

This implies that Type 1 has $b_{n+1} = b_n$ and $z_{n+1} = z_n + d_n$ when $f_n = 0$. Here we set $u_n = 0$, and $h_n = 1$. Type 2 has $b_{n+1} = h_n b_n$ and $z_{n+1} = -f_n + h_n u_n$, when $d_n = 0$. Here we set $u_n = f_n - z_n$, and $h_n = f_n^2/(b_n^2 + u_n^2)$. For processes from right to left, we use $r_n = i c_n + y_n$ in place of q_n , b_n , and z_n . Type 1 has $c_n = c_{n+1}$ and $y_n = y_{n+1} - d_n$ when $f_n = 0$. Here we set $v_n = 0$ and $k_n = 1$. Type 2 has $c_n = k_n c_{n+1}$ and $y_n = f_n - k_n v_n$ when $d_n = 0$. Here we set $v_n = f_n + y_{n+1}$ and $k_n = f_n^2/(c_{n+1}^2 + v_n^2)$. Here u_n , h_n , v_n , and k_n are parameters for convenience in computation.

In the computation, we assume f_n , b_1 , z_1 , c_{10} , and y_{10} are specified. For illustration purposes, we set $d_1 = d_9 = 100$ mm and define $M = (b_1/c_{10})^{1/2}$ as the magnification change necessary to match the input beam at 1 to the output point at 10. The computation is an iteration process, where d_3 and d_7 are set until $|b_5/c_5 - 1|$ is less than 10^{-9} . The condition of self consistency requires that the quantities, $b_5 = (f_2 f_4 / M D_2)^2$ and $c_5 = (f_6 f_8 / D_4)^2$, with the definitions $D_2 = f_2 + f_4 - d_3$ and $D_4 = f_6 + f_8 - d_7$, are equal, since they are the same when the telescope is properly adjusted. We start the iteration process with the control parameter $T = \sqrt{d_7 d_3}$ set to 1200 mm to get a convenient dynamic range for d_7 and d_3 of about 800 mm. Thus, we start $d_7 = M^{1/2}$ and $d_3 = T/M^{1/2}$. The new d_7^N are constrained to be $d_7^N d_3^N = T^2$ where the new d_3^N is related to the old d_3^0 by $d_3^N = d_3^0 [b_5/c_6]^{1/4}$.

We reset $d_7 = d_7^N$ and $d_3 = d_3^N$ for each complete new calculation until $b_5 = c_6$, as desired. When the above is done, then we find $d_5 = y_6 - z_5$.

In this analysis, we provisionally restrict $\frac{1}{2} < M < 1.67$, and $|z_1| < 100,000$ mm as the allowed range of deviation for the laser source beam parameter relative to those of the optical filter. If the source parameters are outside this range, it will be necessary to choose different optics to make the intended measurement sequence. For this magnification range, the range of d_3 is 930 to 1700 mm and d_7 is 850 to 1550 mm. If we design the beam path so that two flat-turning mirrors at 45° angles are used for each distance adjustment, then the travel distance can be 400 mm or less. Under these conditions, the range of d_5 is 2900 to 3100 mm.

The computer program in appendix A generates the values of b_n , z_n , the convenience parameter $p_n = b_n^2 + z_n^2$, the inverse range parameter $e_n = z_n/p_n$, and the beamwidth parameter $w_n = (\lambda p_n/\pi b_n)^{1/2}$ for all n values, as well as the round-trip phase, $z_n - y_n$, discrepancy and the round-trip beamwidth discrepancy $b_n - c_n$. The latter parameters permit a

visual check on the consistency of the computation. In addition, these results are computed with individual changes by 1 mm in d_3 , in d_7 , and in d_5 . Further changes are made of 2 percent in b_1 and 100 mm for z_1 .

This computer program establishes that a telescope can match the phase front and the beamwidth to the optical filter described in section 4. The position adjustments are very sensitive. Thus, they will need accurate control to 1 μm over a 1-cm travel range. The gross adjustment of 400 mm is set at discrete locations of 4 mm spacings. A machined strip of 100 slots provides this accurate spacing. To illustrate the expected results, appendix A has listed a base case with $M = 1$ and $z_1 = -10,000$ mm. No further data will be described since this paper establishes feasibility only.

Reference 4 addresses the effects of spherical aberration, which should be small for the lens with focal length of 2 m. The accuracy of the computer-generated optical filter will be a more significant source of error. If spherical aberration proves to be a problem, we would use two thin lenses with antireflective coatings and glass with an index of 1.5 to reduce both the reflections and the spherical aberrations for 1.06- μm wavelength.

Figure 2.2 shows a top view of how the lens and mirrors could be located to realize a telescope in a space of about 1200 by 1200 by 200 mm³. Figure 2.3 shows a three-dimensional view of the telescope. Notice that the d_9 distance must be greater than the 100 mm designated in the previous discussion because the last mirror which turns the beam out of the plane of the telescope toward the filter requires a space of 200 mm or more. This turning mirror is necessary to make a compact structure of both the filter and the telescope. The optical filter will have a lens of 1 m focal length that is antireflection coated and may be made up of two lenses to make the spherical aberration zero.

3. The Beam Mismatch Between the Telescope and the Ideal Optical Filter

Unless the laser source is a device that has been manufactured with the same physical laser cavity dimensions, and with the same type of other equipment such as the optics, electronics, flash lamp, and the optical resonator for each unit; the beamwidth and phase front from a given unit will not be known without some measurement strategy to establish the proper adjustment of the telescope described in section 2. Therefore, the purpose of this section and appendix B is to indicate the type of formulas needed to define the measurement strategy. It should be obvious that these suggested formulas are complex and will need more information and work as well as a computer to perform the necessary processing of the final results so that the beam mismatch can be minimized properly.

The design issues to allow computer processing are not addressed within this paper. The derived formulas assume that the primary modes of the laser source are represented by a few Hermite modes so that the results are not abstract. If this assumption proves to be false, then it will be necessary to do a new mismatch analysis in the terms of a more appropriate description of the natural modes for the laser source. For now, we do our analysis on the basis that a few Hermite modes characterize the modes of a laser source with sufficient precision.

Using the scalar theory [3], we find that the electric field at the filter can be represented in two ways, namely (1) a complete Hermite expansion showing the coefficients

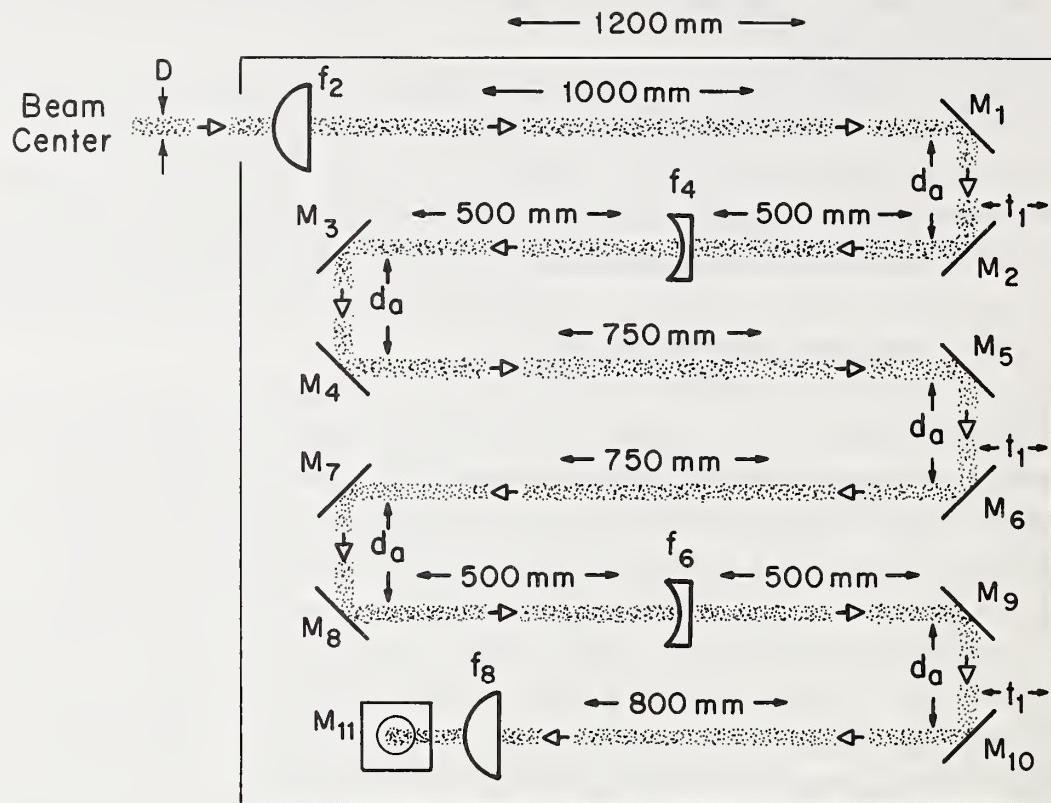


Figure 2.2. Top view of telescope showing nominal dimensions for optics at extreme positions.

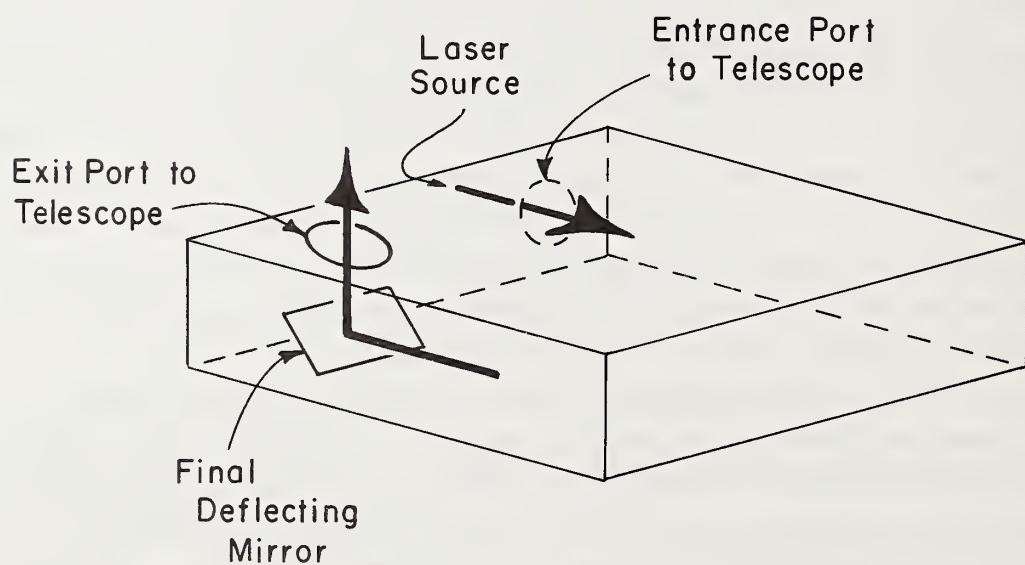


Figure 2.3. Telescope three-dimensional diagram.

that would be measured by an ideal optical filter and (2) a complete Hermite expansion showing six parameters that reflect the change from the ideal match of the optical filter. Equation (3.1) shows the former and eq (3.2) shows the latter. Thus, we have

$$E(x, y, z) = \sum_{n,m} A_{nm} H_n(\sqrt{2x}/w_f) H_m(\sqrt{2y}/w_f) g_0, \quad (3.1)$$

where

$$g_0 = \exp [-(x^2 + y^2)/w_f^2].$$

Here, x and y are the coordinates for the surface of the ideal optical filter. (I use the word *ideal* in this section because the actual filter will have inaccuracies which will be described in section 4.) We assume the z coordinate is zero in this analysis. The w_f is the beamwidth that the ideal filter uses in its decomposition of the electric field. The coefficients $A_{nm} = R_{nm} \exp(i S_{nm})$ have the unknown phase factors, S_{nm} , and the measured amplitudes, R_{nm} .

The electric field is now given in terms of the coordinates of the laser beam in the neighborhood of the ideal optical filter as

$$E(\bar{x}, \bar{y}, \bar{z}) = \sum_{n,m} B_{nm} H_n(\sqrt{2\bar{x}}/w_0) H_m(\sqrt{2\bar{y}}/w_0) g, \quad (3.2)$$

where

$$g = \exp [i k \bar{z} - (\bar{x}^2 + \bar{y}^2) (1/w_0^2 - i k/2R_0)],$$

and where $\bar{x} = -x_0 + x c_1 c_2 - y c_1 s_2$, $\bar{y} = -y_0 + x s_2 + y c_2$, and $\bar{z} = -x s_1 c_2 + y s_1 s_2$. Here $c_1 = \cos \theta_1$, $s_1 = \sin \theta_1$, $c_2 = \cos \theta_2$, and $s_2 = \sin \theta_2$. The θ_1 and θ_2 are the usual spherical coordinates ϕ and ψ , respectively. For this expansion in eq (3.2) to be valid, we assumed w_0 is independent of \bar{z} because θ_1 and θ_2 are small angles of rotation, x_0 , y_0 are small distances for displacement, and the phase mismatch, $r_2 \equiv (k w_f^2 / 2R_0)$ is also small.

It is convenient to define the six parameters of adjustment by using

$$w_0 = w_f(1 + r_1), \quad c_1 = c_2 = 1, \quad s_1 = \theta_1, \quad s_2 = \theta_2, \quad r_2,$$

$$P_0 \equiv \sqrt{2} x_0/w_f, \quad \text{and finally } q_0 \equiv \sqrt{2} y_0/w_f. \quad (3.3)$$

Appendix B shows briefly how the coefficients A_{nm} can be related to the B_{nm} coefficients. Here we note from eqs (B.8) and (B.9) that relationship is in the form

$$A_{nm} = \sum_{n'm'} C_{nn'm'm'} B_{n'm'}, \quad (3.4)$$

where $C_{nn'm'm'}$ can be known by the analysis in appendix B. The actual measurement has

$$R_{nm}^2 = \sum_{n'm'n''m''} C_{nmn'm'} C_{nmn''m''}^* D_{n'm'n''m''}. \quad (3.5)$$

Here

$$D_{n'm'n''m''} \equiv \langle B_{n'm'} B_{n''m''}^* \rangle. \quad (3.6)$$

The brackets represent some form of averaging of the phase from each mode of the laser source. If the laser has truly Hermite modes, then a reasonable hypothesis is

$$D_{n'm'n''m''} = K_{nm}^2 \delta_{n''}^{n'} \delta_{m''}^{m'}. \quad (3.7)$$

This means that each mode has a uniform random phase relationship to the other modes. Other hypotheses will be necessary if the basis modes of the laser source are not these Hermite modes but are some linear combination of them which then can be treated as containing random phase relationships between the new modes (see section 5 on this point). Additional information is needed to complete this mismatch analysis. At this time we do not have that information, therefore we stop the discussion here.

In summary, we have just shown the type of analysis necessary to unravel the measurements of R_{nm}^2 from the actual mode strength, namely, the B_{nm} set. Various testing will have to be done to determine what is the appropriate assumptions on the character of $D_{nmn'm'}$ for each type of laser source system.

4. Design of the Optical Filter Unit

In section 4.1 we develop an almost orthonormal set of over-complete functions that can be used to understand the detector system, the ideal filter, and finally the nonideal filter. In addition, some discussion is made in appendix C to identify how this set can be changed to better reflect what happens in a laser source.

In section 4.2 we develop the formulation to simulate how the detector unit can be represented and how it can operate to measure the individual modes resulting in a prescribed pattern in the focal plane of the lens. We also define the conditions that the detectors must satisfy.

In section 4.3 we set up the ideal optical filter analysis using the Hermite functions as the modes.

In section 4.4 we do the nonideal filter analysis to determine the problems in realizing an optical filter. Here we learn what assumptions and procedures are necessary to get the appropriate mode analysis.

Section 4.5 summarizes the results of these sections. The appendices D, E, F, and G have computer programs used to check the ideal and nonideal filter analysis. They are referenced in the appropriate subsections.

Figure 4.1 shows a side view of the optical filter unit. The tube contains a lens of 1000 mm focal length which is coated to minimize the effects of reflection at 1.06 μm wavelength. There is a filter plate located in front of the lens (illustrated in fig. 4.2).

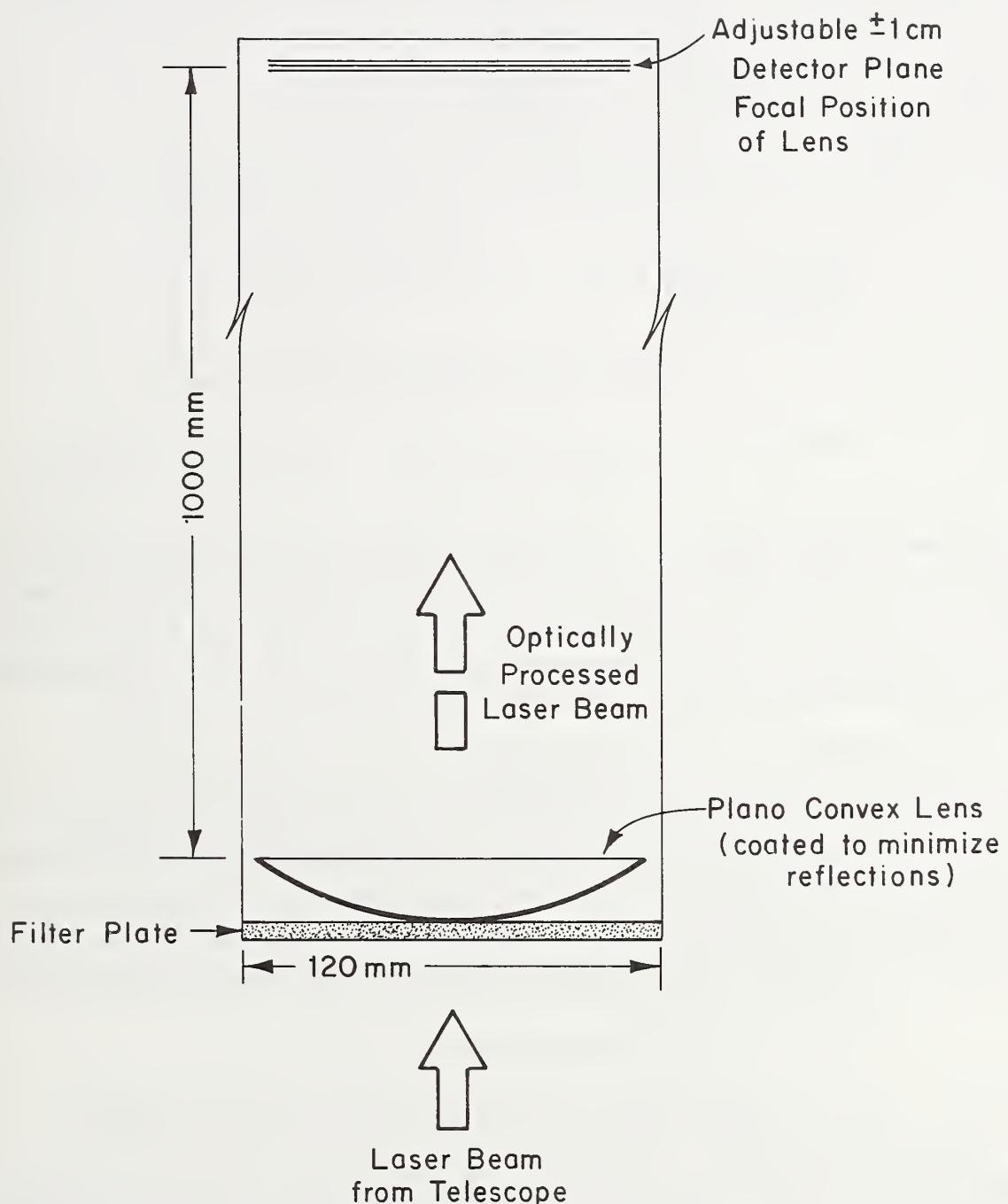


Figure 4.1. Side view of optical filter unit showing functions and nominal dimensions.

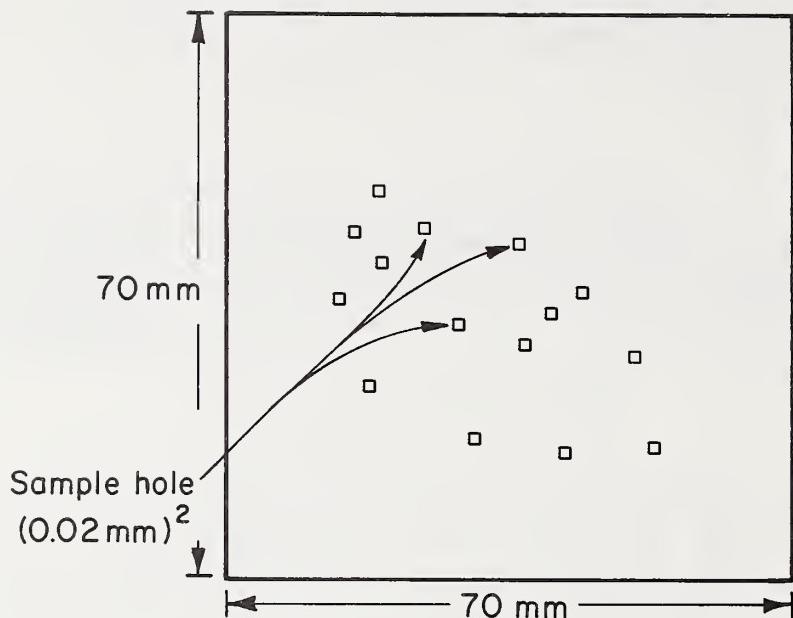


Figure 4.2. Filter plate (front view). Illustration of size and type of sample holes. Actual pattern is complex.

The device performs the optical computing on the incident laser beam from the telescope. There is an array of detectors at the end of the tube located in the focal plane of the lens. This array maybe a CC device or a TV scanning unit that has uniform response at the 1.06 μm wavelength. The filter is a complex pattern of grid holes which samples the incoming beam and causes both phase and amplitude modulation of that beam.

4.1 The Basic Functions for Optical Computing

We define the following as one possible set of basis functions that may be used in optical computing. This set is not unique. Others are possible and should be considered when the exact conditions in the laser source are known. For this study, we assume (1) that the low order ($n < 5$) Hermite modes contain 95 percent of the laser energy, and (2) that the detectors are placed at equal spacing in the focal plane of the lens. Thus, we define

$$H(n, k, x) \equiv U(n) H_n(\sqrt{2} x / w_f) \exp [-(x/w_a)^2] * [\cos(2\pi k f x) (1 + (-1)^n)/2 + \sin(2\pi k f x) (1 - (-1)^n)/2], \quad (4.1)$$

where $U(n) \equiv [\sqrt{2} c N / (\sqrt{\pi} w_f n! 2^n)]^{1/2}$, and where $k \geq 0$ only. The c is the spacing of the grid holes in the nonideal filter. The $c = 0.02 \text{ mm}$ for our test. N is the number of grid holes in the range $0 \leq x \leq L$. Here, $L = 35 \text{ mm}$ and, hence, $N = 1750$. The w_f is the filter beamwidth which is 10 mm for our design. The $f = d/\lambda z$, where d is the distance between detector elements, z is the focal length of the lens, and $\lambda = 1.06 \mu\text{m}$ the wavelength. The $d = 10 w_s$, where w_s is the Gaussian sampling width of the detector element. Finally, the w_a is defined by

$$\frac{1}{w_a^2} \equiv \frac{1}{w_f^2} + \frac{1}{w_c^2},$$

where $w_c = \lambda z/(w_s)$, and w_s is set equal to 0.02 mm. It is a coincidence that w_s and c are the same. It represents the type of dimensions that can be realized for fabrication elements in both detectors and in grid holes of the film. If we define a weight function,

$$W(x) = \exp [2 (x/w_c)^2], \quad (4.2)$$

and allow an integration over the range $-\infty < x < +\infty$, the following results are approximately true to a part in 10^{-8} or better for all n and $k \geq 0$. Thus,

$$\int_{-\infty}^{\infty} dx H(n, k, x) W(x) H(n_1, k_1) = \delta_{k_1}^k \delta_{n_1}^m F, \quad (4.3)$$

where

$$F = cN[1 + \sum_0^k (-1)^n],$$

and $\delta_{k_1}^k = 0$ unless $k = k_1$.

If the range of integration is $\pm L$ instead of $\pm \infty$, then the result of eq (4.3) is not exactly true. For the low-order Hermite modes, namely $n \leq 4$, the computer results show that it is valid. Also, for low-order modulation of $k \leq 8$, the computer results show that it is valid. If the final system tends to use $n \geq 5$, then it becomes necessary to develop a different basis that reflects the truncation at $x = L$. Appendix C suggests a way this can be done.

There are two features of this basis set, namely, the functions are symmetric in x , $H(n, k, x) = H(n, k, -x)$, and we assume that the y coordinate has the same basis functions, thus $H(n, k, y)$. If we assume the changes in $H(n, k, x)$ are small compared to the grid size, then we can make the discrete structure with $x_l = (l+1/2)c$ where $0 \leq l \leq N-1$, and we can presume that the basis filter design can be covered with the range of x being $0 \leq x \leq L$ and y being $0 \leq y \leq L$. The other quadrants can use the same grid structure in the fabrication of the filter. Thus, we have the filter function described with the aid of the function $T(x)$ as

$$T(x) \equiv \sum_{n=0}^5 \sum_{k=0}^3 H(n, k, x) S(n, k) W(x), \quad (4.4)$$

where $T(x) > 0$ and $T(-x) = T(x)$. We cut off the n and k in our computer simulation work, a decision reflected in eq (4.4). The y -coordinate function has the same form. The total filter can be described as

$$T(x, y) = T(x) T(y)$$

for this analysis. This assumption of form allows separation of the two dimensions and extensive simplification in the structure of the computer-generated models. If this proves to be invalid, then the problem requires significantly more speed in computation. For this

paper, we have assumed these conditions so that we can define what will be the problems in the eventual fabrication of the nonideal optical filter. The nonseparability of coordinates can then be addressed as necessary in a later study.

The discrete function analysis gives us the result

$$\sum_{\ell=0}^{N-1} H(n, k, x_\ell) W(x_\ell) H(n_1, k_1, x_\ell) \equiv \delta_{k_1}^k \delta_{n_1}^n \frac{N}{2} (1 + \delta_0^k (-1)^n)$$

for the range of $0 \leq k < 9$ and $0 \leq n \leq 5$. Appendix D has the computer programs that generate the $H(n, k, x_\ell)$ set. Appendix F has the computer program that selects the strength of the set used in eq (4.4) by determining the value of $S(n, k)$ for the nonideal case. We discuss this situation in more detail in section 4.4.

We have a set of basis functions which can be used provided the modes $n > 5$ are not present in the primary beam. If they are present, then the procedures indicated in appendix C become necessary. For the rest of this section we assume the energy content of the laser source is such that only the $n \leq 5$ is true for both the x and y coordinates. Using reference 5 we have the electrical field at the optical filter given as

$$U_1(x, y) = \sum_{n,m=0}^5 A_{n,m} U(n) U(m) H_n(\sqrt{2} x/w_f) H_m(\sqrt{2} y/w_f) G(x) G(y),$$

where

$$G(x) \equiv \exp[-x^2/w_f^2]. \quad (4.5)$$

The electrical field at the detectors is given as

$$V_2(x_1, y_1) = \frac{1}{i\lambda z} \int dx dy T(x, y) U_1(x, y) \exp\left[\frac{-2\pi i}{\lambda z}(xx_1 + yy_1)\right]. \quad (4.6)$$

By using the separability assumption for the filter, we construct the following form for U_2 ,

$$U_2(x_1, y_1) = \frac{1}{i\lambda z} \sum_{n,m} A_{n,m} V_n(x_1) V_m(y_1) w_f^2 \frac{H}{2}$$

with

$$V_n(x_1) = \frac{\sqrt{2}}{w_f \sqrt{\pi}} \int dx U(n) H_n(\sqrt{2} x/w_f) T(x) \exp[-(x/w_f)^2] \exp[-2\pi i xx_1/\lambda z]. \quad (4.7)$$

These $V_n(x_1)$ are convenient to simulate the detector.

4.2 The Detector Analysis

We look at $V_n(x_1)$ and form a convenient strategy for evaluation of the finite spot size for the detector. We assume that each detector has a spatial response that is Gaussian; thus, we convolve the response function to define an effective detector response at position "j" as

$$K(n,j) = \frac{1}{w_s \sqrt{\pi}} \int dx_1 V_n(x_1) \exp [-(x_1 - jd)^2 / w_s^2]. \quad (4.8)$$

This form is convenient for the computer simulations and adequately represents what happens at a given detector. In TV-type scan systems it may prove necessary to develop a weighting algorithm for the response from each of the detector elements in the chip to simulate a Gaussian response profile.

If we use eq (4.7) and integrate over x_1 , make the definition

$$F(n,j) \equiv S(n,j) [1 + \sum_0^j (-1)^n] = \int_{-\infty}^{\infty} dx T(x) H(n,j,x)/Nc,$$

and note that $K(n,-j) = K(n,j)$ for n even, and $K(n,-j) = -K(n,j)$ for n odd; we get for $j \geq 0$,

$$K(n,j) = F(n,j) [Nc\sqrt{2}/\sqrt{\pi}w_f] * [(1+(-1)^n) - i(1-(-1)^n)]/2. \quad (4.9)$$

We can identify the set of Fourier-type transform relationships as

$$S(n,j) [1 + \sum_0^j (-1)^n] = \frac{2}{Nc} \int_0^{\infty} dx T(x) H(n,j,x) \quad (4.10)$$

and eq (4.4).

We can further modify this structure to reflect the fact that our analysis is over a finite range with discrete elements. Thus, we get

$$T(x_\ell) = \sum_{m=0}^5 \sum_{k=0}^9 H(m,k,x_\ell) S(m,k) W(x_\ell)$$

and

$$S(n,k) [1 + \sum_0^k (-1)^n] = \frac{2}{N} \sum_{\ell=0}^{N-1} T(x_\ell) H(n,j,x_\ell). \quad (4.11)$$

4.3 The Ideal Filter Characteristics

The ideal filter has $S(n,k) = 0$ except for those cases shown in table 4.1. The relative size of each coefficient $R(0)$ to $R(5)$ was chosen to maximize the strength of the individual modes for each Hermite structure as well as allow separation between the modes. The $R(6)$ to $R(9)$ were chosen to allow the $T(x)$ be positive definite for all x . A single scale factor remains by which each coefficient can be multiplied. These coefficients allow $T(x)$ to be less than 1.04 for all x . This scale value was chosen to minimize the error of the nonideal filter. Its value should be 1.00. For the purpose of this report, the explicit scale change is unnecessary. Thus, to get the correct $R(0)$, etc. for the ideal filter divide the values given by $A = 1.04$. The programs D, E, and G are used to generate the

Table 4.1. The Strengths of the Ideal Filter
(A = 1.04)

S(0,0) = 0.116	= R(6) * A
S(2,0) = 0.0408	= R(7) * A
S(4,0) = 0.04	= R(8) * A
S(6,0) = 0.04	= R(9) * A
S(0,2) = 0.01687944	= R(0) * A
S(1,3) = 0.022859717	= R(1) * A
S(2,4) = 0.023948275	= R(2) * A
S(3,5) = 0.027209995	= R(3) * A
S(4,6) = 0.027589743	= R(3) * A
S(5,7) = 0.029858334	= R(5) * A

Table 4.2. The Ideal Filter Mode Mixing
S(k,n) Elements

n=	k	Pure	0	1	2	3	4	5
2	1	-8.9E-7		-4.1E-7		+1.4E-5		+5.3E-5
3	-3.7E-7	1		1.4E-5		4.9E-5		-1.3E-4
4	-2.4E-6	1.3E-5		1		-1.5E-4		-4.6E-4
5	6.2E-6	-3.7E-5		-1.2E-4		1		8.8E-4
6	2.2E-5	-8.4E-5		-3.9E-4		8.6E-4		-4.5E-3
7	6.3E-6	3.2E-5		-1.2E-4		-3.5E-4		9.1E-4
								1

Table 4.3. The Mode Mixing for the Nonideal
Filter with a Two-Level Quantization

1	+1.2E-3	-1.7E-2	-6.0E-3	-6.8E-5	5.7E-4
+1.8E-2	1	-1.1E-2	-1.3E-2	+5.1E-3	2.6E-3
-7.1E-3	-2.8E-3	+1	-2.6E-2	1.8E-2	2.0E-2
5.1E-3	6.5E-3	1.2E-2	1	4.8E-3	-8.1E-4
1.4E-2	1.1E-2	-4.3E-2	2.4E-2	1	-3.5E-2
7.7E-2	1.9E-2	-5.0E-2	-2.0E-2	-2.2E-2	1

expected self consistency for $S(n,k)$, given the values used to generate the $T(x)$ from table 4.1. The basic conclusion is that mode mixing is less than 0.45 percent and this is primarily due to the fact that the $n = 5$ mode has truncation error. It is possible to eliminate this mixing by suitable orthogonality procedures. This would be equivalent to a rigorous account of the finite range of each mode, namely $L = 35$ mm. I did not do this in this report since the prime emphasis here is to identify the key areas of error and to try to devise a technique for elimination or reduction of those errors. The fabrication of the nonideal filter completely masks these errors so they are ignored at this time. Once a technique dealing with the errors generated by the nonideal filter has been determined, then it may prove worthwhile to do the orthogonality. Finally, the choice of order and strength for each mode has been arbitrary, it may prove necessary to define other orders and strengths to minimize cross correlation effects in nonideal filters.

4.4 The Nonideal Filter Characteristics

The nonideal filter has one modification from the ideal filter where $T(x)$ ranges from the values 0 to 1 and has $T(x)$ either 0 or 1 at each cell location. This implies either 0 percent or 100 percent transmission. It is technically possible to construct quantized filters with three levels, namely 0 percent, 50 percent, or 100 percent and so forth. However, it is fairly easy to fabricate a filter of 100 percent or 0 percent transmission; therefore, it is desirable to consider such a nonideal filter. Using the program "Check" as shown in appendix G with the nonideal filter generated by the program in appendix E, we see in table 4.3 a mixing between the modes of up to the 3 percent level. If we assume that the phase correlation between modes is random, then the mode mixing has an error of about 1 percent for the determination of the energy in each mode. If there is no phase averaging, then it becomes necessary to do one of two steps; consider a new set of modes which can have a random phase relationship between the modes or improve the nonideal filter by using a three or more level quantization process instead of the two level quantization. In the latter case, we accept that the problems of fabrication, such as variable thickness transmission films, can be solved economically or that the cell size $c = 0.02$ can be changed to $c = 0.02*S$ and the number of elements becomes $N' = N/S$. Here, $S = 3$ allows a four-level gray scale. The basic two-level quantization is done in "Select" by the requirement

$$Y(x_{\ell_1}) \equiv \sum_{\ell=\ell_1}^N T(x_{\ell}) - \sum_{\ell=\ell_1+1}^N E(x_{\ell}), \quad (4.12)$$

which is a running sum. Here $E(x_{\ell}) = 1$ whenever $Y(x_{\ell}) > 0.5$, otherwise $E(x_{\ell}) = 0$. It is not known at this time if this is the best possible two-level scheme for quantization. All attempts to improve the scheme have failed because the rate of computation has been too slow. New time-efficient computation strategies are necessary.

4.5 Summary of Results on the Optical Filter

Briefly, this section has demonstrated that it is possible to devise an optical filter that can do direct mode analysis of laser beams. There are some technical difficulties that

still need further work to make the construction of these filters as efficient as possible. Appendix C suggests what needs to be done is devise the most appropriate modes for a particular laser source. The comparison of the results for two-level quantization with the nonquantized version suggests that it is very desirable to devise a means to fabricate at least a three or more level quantization process and/or devise a time-efficient strategy for minimizing the consequence of the quantization process.

5. Conclusions

We have learned the following:

1. It is possible to construct a telescope which will allow both a variation in the beam-width and the adjustment of the phase-front curvature. This unit is necessary only if the laser source has a beamwidth and phase front that vary significantly compared to the final desired accuracy from the quality assurance measurements on a particular laser source. It is necessary to build a matching telescope for a laboratory beam-profile measuring apparatus so that each source can be assessed to determine if the source beam profile can be evaluated adequately and to establish what the need is for such a telescope in a field instrument.
2. The nonquantized version of an optical filter can perform mode analysis with separation of better than 0.1 percent of the amplitudes. At least one such device would be of great value to allow a determination of the Hermite mode structure for various laser sources. Once these structures are known, it is possible to devise optical filters which do not require telescopes, which require only a few detectors, which require no extensive computation, and hence can be devised to allow construction of a GO-NO-GO type measurement system. The latter unit will be valuable for field sites using personnel who have very little training. Technical realization of the nonquantized optical filter would be done with multiple exposures of high resolution (3000 lines/mm or better) film.
3. The two level quantization process produces extensive mode mixing (about 8 percent). To make a quantized optical filter useful, a new condition becomes important. A mode analysis other than Hermite may become appropriate. These new modes are defined by the condition that the phase relationship between modes is random. This condition will allow mode separation to better than 1 percent which is more than adequate accuracy for a GO-NO-GO type instrument.
4. If the particular laser source has no stable mode structure, then it becomes necessary to use an artificial mode set such as the truncated Hermite modes in conjunction with a quantized process that is more than two level. In this way we can construct a GO-NO-GO measuring device.

In summary, this paper has demonstrated that the optical processing can be used to do mode analysis and that the optical filter must be designed uniquely for each generic laser source (instrument). In addition, it can be concluded that fast computer techniques need to be developed to devise the optimum-quantized optical filter for two dimensional beam patterns without any significant symmetries.

The following persons have been helpful toward completing the design of this system: Don Boyle and William Hall have helped by allowing me to use their computer system to perform the simulations and by providing consultation in the use of the machine. William Hall has provided the necessary skills to get the final program converted into FORTRAN from Basic so that it would work 30 times faster and thereby will allow me to look at level quantizations. Aaron Sanders and Matt Young have been helpful as active sounding boards to react to the numerous ideas that were eliminated and to the ideas that resulted in the formulation presented in this paper. Edie DeWeese and Susie A. Rivera did the excellent typing and composition work on the paper. Dean Eicher supplied the fine drawings.

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6. References

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Appendix A. Telescope

```

10 REM THIS PROGRAM SETS UP THE TELESCOPE FOR BEAM MEASUREMENTS
20 CLOSE 1
30 DIM X(10,2)
40 OPEN "CEN:",1,1
50 PRINT "NUMBER OF W(1),Z(1) PAIRS"
60 INPUT N
70 FOR I=1 TO N
80 PRINT "READ IN PAIRS"
90 INPUT X(I,1),X(I,2)
100 NEXT I
110 FOR I=N+1 TO 10
120 X(I,1)=0
130 NEXT I
140 K8=1
150 DIM B(10),Z(10),C(10),Y(10),U(10),H(10),F(10),K(10),V(10),D(10)
160 DIM W(10)
170 P9=3.14159
180 DATA 0,2000,0,-2000,0
190 DATA -2000,0,2000,0,2000
200 W(1)=X(K8,1):Z(1)=X(K8,2)
210 K8=K8+1
220 IF K8=11 GOTO 1530
230 A=.1064E-2
240 B(1)=P9*W(1)^2/A
250 IF W(1)=0 GOTO 1530
260 W(10)=10
270 C(10)=P9*W(10)^2/A
280 M=SQR(B(1)/C(10))
290 M1=SQR(M)
300 T=1200
305 RESTORE
310 FOR I=1 TO 10
320 READ F(I)
330 IF F(I)=0 GOTO 360
340 D(I)=0
350 GOTO 400
360 U(I)=0
370 V(I)=0
380 H(I)=1
390 K(I)=1
400 NEXT I
410 D(1)=100
420 D(9)=100
430 D(7)=M1*T
440 D(3)=T/111
450 FOR I=1 TO 4
460 GOSUB 750
470 NEXT I
480 FOR J=1 TO 4
490 I=10-J
500 GOSUB 860
510 NEXT J
520 N3=0
530 B1=B(5)/C(6)
540 B2=B1-1
550 IF ABS(B2)<.1E-3 GOTO 670
560 N3=N3+1: IF N3>6 GOTO 1550
570 B3=SQR(B1)
580 B4=SQR(B3)
590 D(3)=B4*D(3)
600 D(7)=D(7)/B4
610 PRINT "CONVERGENCE VALUE B2=",B2
620 I=3: GOSUB 750
630 I=4: GOSUB 750
640 I=7: GOSUB 860
650 I=6: GOSUB 860

```

```

650 GOTO 530
670 D(5)=Y(6)-Z(5)
680 FOR K7=1 TO 7
690 REM TEST GENERATION DATA
700 N8=K7
710 GOSUB 1390
720 NEXT K7
730 PRINT "CURRENT RUN CASE. K8=",K8
740 GOTO 200
750 REM SUBROUTINE FOR LEFT TO RIGHT GENERATION
760 I1=I+1
770 IF F(I)=0 GOTO 830
780 U(I)=F(I)-Z(I)
790 H(I)=F(I)↑2/(B(I)↑2+U(I)↑2)
800 B(I1)=H(I)*B(I)
810 Z(I1)=-F(I)+H(I)*U(I)
820 GOTO 850
830 B(I1)=B(I)
840 Z(I1)=Z(I)+D(I)
850 RETURN
850 REM SUBROUTINE FOR RIGHT TO LEFT GENERATION
870 I1=I+1
880 IF F(I)=0 GOTO 940
890 V(I)=F(I)+Y(I1)
900 K(I)=F(I)↑2/(C(I1)↑2+V(I)↑2)
910 C(I)=K(I)*C(I1)
920 Y(I)=F(I)-K(I)*V(I)
930 GOTO 960
940 C(I)=C(I1)
950 Y(I)=Y(I1)-D(I)
960 RETURN
970 REM SUBROUTINE TO GENERATE LEFT AND RIGHT BEAM TRACE
980 FOR I=1 TO 9
990 GOSUB 750
1000 NEXT I
1010 FOR J=1 TO 9
1020 I=10-J
1030 GOSUB 860
1040 NEXT J
1050 RETURN
1060 REM PRINT BLOCK NUMBER 1
1070 PRINT ON (1)"TELESCOPE PROGRAM , VERSION 2/27/81"
1080 PRINT ON (1)GETDATES(0),"DATE OF RUN HERE"
1090 PRINT ON (1)GETTIMES(0),"TIME OF RUN HERE"
1100 PRINT ON (1)"FIXED VARIABLES ARE IN MM UNITS. THEY ARE"
1110 PRINT ON (1)"D(1)=",D(1)
1120 PRINT ON (1)"D(9)=",D(9)
1130 PRINT ON (1)"W(10)=",W(10)
1140 PRINT ON (1)"T IS THE DISTANCE CONSTRAINT FOR D(3),D(7)"
1150 PRINT ON (1)"T=",T
1160 PRINT ON (1)"THE MAGNIFICATION SET FOR TELESCOPE IS ",M
1170 PRINT ON (1)"THE ACTUAL MAGNIFICATION IS "
1180 M8=SQR(B(1)/C(10))
1190 PRINT ON (1)"M8=",M8
1200 RETURN
1210 REM PRINT BLOCK NUMBER 2
1220 FOR I=1 TO 10
1230 PRINT ON (1)"B      ,Z      ,D      ,F      FOR I=",I
1240 P=B(I)↑2+Z(I)↑2:E=Z(I).P
1250 W1=SQR(A*P/(P9*B(I)))
1260 PRINT ON (1),B(I),Z(I).D(I),F(I)
1270 PRINT ON (1)"INVERSE RANGE, WIDTH      , PHASE DIFFER,    DELTA B "
1280 PRINT ON (1) E,W1,Z(I)-Y(I),B(I)-C(I)
1290 NEXT I
1300 RETURN
1310 PRINT ON (1)"BASE DATA CASE": RETURN
1320 PRINT ON (1)"CHANGE D(3)": RETURN
1330 PRINT ON (1)"CHANGE D(5)": RETURN

```

```
1340 PRINT ON (1)"CHANGE D(?)": RETURN
1350 PRINT ON (1)"CHANGE B(1)": RETURN
1360 PRINT ON (1)"CHANGE Z(1)": RETURN
1370 PRINT ON (1)"BASE DATA CASE REPEAT ": RETURN
1380 REM SUBROUTINE FOR GENERATING THE TEST CHANGES
1390 ON N8 GOSUB 1450,1470,1480,1490,1500,1510,1520
1400 GOSUB 970
1410 GOSUB 1060
1420 ON N8 GOSUB 1310,1320,1330,1340,1350,1360,1370
1430 GOSUB 1210
1440 RETURN
1450 D3=D(3): REM BASE DATA FOR SAVING
1460 D5=D(5):D7=D(7):B8=B(1):Z8=Z(1): RETURN
1470 D(3)=D3+1: RETURN
1480 D(3)=D3:D(5)=D5+1: RETURRN
1490 D(5)=D5:D(7)=D7+1: RETURN
1500 D(?)=D7:B(1)=.98*B8: RETURN
1510 B(1)=B8:Z(1)=100+Z8: RETURN
1520 Z(1)=Z8: RETURN
1530 CLOSE 1
1540 STOP
1550 PRINT "NO CONVERGENCE"
1560 END
```

TELESCOPE PROGRAM , VERSION 2/27/81
—03/02/81— DATE OF RUN HERE

09:44:07 TIME OF RUN HERE
 FIXED VARIABLES ARE IN MM UNITS. THEY ARE
 D(1)= 100
 D(9)= 100
 W(19)= 10
 T IS THE DISTANCE CONSTRAINT FOR D(3),D(7)
 T= 1200
 THE MAGNIFICATION SET FOR TELESCOPE IS 1
 THE ACTUAL MAGNIFICATION IS
 M3= 1
 BASE DATA CASE
 B Z ,D ,F FOR I= 1
 295262.22 -100000 100 0
 INVERSE RANGE, WIDTH , PHASE DIFFER, DELTA B
 -.10290207E-5 10.557951 -.15955455E-3 .19515411E-3
 B Z ,D ,F FOR I= 2
 295262.22 -99900 0 -2000
 INVERSE RANGE, WIDTH , PHASE DIFFER, DELTA B
 -.10232032E-5 10.555876 -.15955455E-3 .19515411E-3
 B Z ,D ,F FOR I= 3
 12.105452 -1995.8222 1164.5814 0
 INVERSE RANGE, WIDTH , PHASE DIFFER, DELTA B
 -.5910232E-3 10.555876 -.21424285E-9 -.1033268E-7
 B Z ,D ,F FOR I= 4
 12.105452 -331.14083 0 -2000
 INVERSE RANGE, WIDTH , PHASE DIFFER, DELTA B
 -.12029104E-2 4.396594 .21424285E-9 -.1033268E-7
 B Z ,D ,F FOR I= 5
 35.438068 -1421.7732 2556.3829 0
 INVERSE RANGE, WIDTH , PHASE DIFFER, DELTA B
 -.70291035E-3 4.396594 0 -.30254871E-7
 B Z ,D ,F FOR I= 6
 35.438068 1234.5097 0 -2000

INVERSE RANGE, WIDTH 1 , PHASE DIFFER, DELTA B
 .89930578E-3 3.8182958 0 .30254871E-7
 B ,Z ,D ,F FOR I= 7
 13.545719 763.52317 1235.3897 0
 INVERSE RANGE, WIDTH 4 , PHASE DIFFER, DELTA B
 .13093058E-2 3.8182958 -.25329427E-9 -.11562591E-7
 B ,Z ,D ,F FOR I= 8
 13.545719 1999.9128 0 2000
 INVERSE RANGE, WIDTH 1 , PHASE DIFFER, DELTA B
 .49999985E-3 10.00001 -.25329427E-9 -.11562591E-7
 B ,Z ,D ,F FOR I= 9
 295262.22 -99.99977 100 0
 INVERSE RANGE, WIDTH 1 , PHASE DIFFER, DELTA B
 .11470522E-8 10.00001 .87630897E-5 .25192453E-3
 B ,Z ,D ,F FOR I= 10
 295262.22 .22954695E-3 0 2000
 INVERSE RANGE, WIDTH 4 , PHASE DIFFER, DELTA B
 .25330287E-14 10 .87630897E-5 .25192453E-3

Appendix B. Mode Mismatch

Reference 6 shows the form for generating the Hermite modes, namely

$$G = \exp[2pu - u^2] = \sum_{n=0}^{\infty} H_n(p) u^n / n! \quad (B.1)$$

Thus, $H_n(p) = (\partial_p^n G|_{u=0})$. This means we differentiate the generator, G , n times and then set $u = 0$ to identify the $H_n(p)$ mode.

Reference 6 also shows the normalization of Hermite function under the integration. Thus, we have

$$\int_{-\infty}^{\infty} dx \sqrt{2/(w_f \sqrt{\pi})} H_n(\sqrt{2}x/w_f) H_m(\sqrt{2}x/w_f) \exp(-2x^2/w_f^2) = \delta_m^n 2^n n! \quad (B.2)$$

The eqs (3.1) and (3.2) can be rewritten in terms of the generator eq (B.1) as (suppress unnecessary variables)

$$\begin{aligned} E(x, y) &= \sum_{n_1 n_2} (\partial_{\mu_3})^{n_1} (\partial_{\mu_4})^{n_2} g B_{n_1 n_2} \exp[-u_3^2 - u_4^2 + 2\frac{\sqrt{2}}{w_0} (\bar{x}u_3 + \bar{y}u_4)] \\ &= \sum_{n_1 n_2} (\partial_{\mu_3})^{n_1} (\partial_{\mu_4})^{n_2} g_0 A_{n_1 n_2} \exp[-\mu_3^2 - \mu_4^2 + 2\frac{\sqrt{2}}{w_f} (x\mu_3 + y\mu_4)]. \end{aligned} \quad (B.3)$$

We define

$$w \equiv \int \frac{2dxdy}{\pi w_f^2} \exp(S), \quad (B.4)$$

with

$$\begin{aligned} S &= ik\bar{z} - (x^2 + y^2)/w_f^2 - (u_1^2 + u_2^2 + u_3^2 + u_4^2) \\ &\quad - (\bar{x}^2 + \bar{y}^2)(1/(1+r_1)^2 - i2r_2)/w_f^2 \\ &\quad + (2\sqrt{2}/w_f)[u_1x + u_2y + (\bar{x}u_3 + \bar{y}u_4)/(1+r_1)]. \end{aligned} \quad (B.5)$$

We can apply eq (B.2) by multiplying eq (B.4) and by integrating the resulting equation. The results are

$$A_{nm} = \sqrt{c_1} \sum_{n_1 n_2} (\partial_{\mu_1})^{n_1} (\partial_{\mu_2})^{m_1} (\partial_{\mu_3})^{n_1} (\partial_{\mu_4})^{m_1} B_{n_1 m_1} W/D_{nm}, \quad (B.6)$$

with $D_{nm} = (2)^{n+m} n! m!$. After a straightforward but tedious integration over x and y in the W , we find W takes the form, $W = W_0 W_1$, where we have the many definitions given below:

$$\begin{aligned}
W_0 &= \exp(g_S), \quad g_S = 2(u_2u_4 + u_1u_3) \\
W_1 &= \exp(g_r)/(1+f_7)^{1/2} \\
f_7 &= f_1 + f_2 + f_1f_2 - f_7^2/4 \\
g_r &= -f_6 - (u_1+u_3)^2 f_1/(1+f_1) - (u_2+u_4)^2 f_8/(1+f_8) \\
&\quad + [2 f_4(u_1+u_3) + f_4^2]/(1+f_1) \\
&\quad + 2(u_2+u_4)[f_5 + 2(u_1+u_3)f_3/(1+f_1)]/(1+f_8) \\
&\quad + [f_5 + 2(u_1+u_3)f_3/(1+f_1)]^2/(1+f_8) \\
f_8 &= f_2 - f_3^2/[4(1+f_1)],
\end{aligned}$$

here

$$\begin{aligned}
f_1 &\equiv (\frac{1}{2(1+r_1)^2} - \underline{i}r_2)(c_1^2c_2^2 + s_2^2) - \frac{1}{2}, \\
f_2 &\equiv (\frac{1}{2(1+r_1)^2} - \underline{i}r_2)(c_2^2 + c_1^2s_2^2) - \frac{1}{2}, \\
f_3 &\equiv (\frac{1}{2(1+r_1)^2} - 2\underline{i}r_2)(s_2c_2)s_1^2, \\
2f_4 &\equiv 2[\frac{c_1c_2}{(1+r_1)} - 1]u_3 + \frac{2}{1+r_1}u_4s_2 - \frac{i}{\sqrt{2}}\frac{kw_f}{f}s_1c_2 \\
&\quad + 2(\frac{1}{2(1+r_1)^2} - \underline{i}r_2)(p_0c_1c_2 + q_0s_2), \\
2f_5 &= 2u_4(\frac{c_2}{1+r_1} - 1) - 2\frac{u_3c_1s_2}{1+r_1} + \frac{ik}{\sqrt{2}}w_f s_1s_2 \\
&\quad + 2(\frac{1}{2(1+r_1)^2} - \underline{i}r_2)(q_0c_2 - p_0c_1s_2) \\
f_6 &\equiv \frac{2}{(1+r_1)}(u_3p_0 + u_4q_0) + (\frac{1}{2(1+r_1)^2} - \underline{i}r_2)(p_0^2 + q_0^2). \tag{B.7}
\end{aligned}$$

We can eliminate W_0 in eq (B.6) to get

$$D_{nm} A_{nm} = \sum_{n'm'} B_{n'm'} (\partial_{\mu_3})^{n'} (\partial_{\mu_4})^{m'} (\partial_{\mu_1} + 2u_3)^n (\partial_{\mu_2} + 2u_4)^m W_1. \tag{B.8}$$

To go further we would assume the fact that θ_1 , θ_2 , r_1 , r_2 , P_0 , and q_0 are small and consistently retain only the necessary terms. For our purposes this exercise is unnecessary; therefore, we stop our analysis and just show the definition of $C_{nmn'm'}$ as

$$C_{nmn'm'} = \frac{1}{D_{nm}} (\partial_{\mu_3})^{n'} (\partial_{\mu_4})^{m'} (\partial_{\mu_1} + 2u_3)^n (\partial_{\mu_2} + 2u_4)^m w_1. \quad (B.9)$$

Appendix C. The Unseparated Case Analysis

The new formulas of similar form to text that would be used in the more complicated analysis are given as:

$$U_1(x, y) = \sum_n A_n H_n(x, y)$$

and

$$U_2(x_1, y_1) = \frac{1}{i\lambda z} \sum_n A_n V_n(x_1, y_1) (Nc)^2. \quad (C.1)$$

Here we define

$$K(j_1, j_2, n) = \frac{1}{w_s^2 \pi} \int dx_1 dy_1 V_n(x_1, y_1) G[j_1, j_2, x_1, y_1],$$

where

$$G(j_1, j_2, x_1, y_1) \equiv \exp \left[-\frac{1}{w_s^2} \{(x_1 - j_1 d)^2 + (y_1 - j_2 d)^2\} \right], \quad (C.2)$$

and

$$V_n(x_1, y_1) \equiv \int dx dy H_n(x, y) T(x, y) \exp \left[-\frac{i2\pi}{\lambda z} (x_1 x + y_1 y) \right] / (Nc)^2. \quad (C.3)$$

The filter function has the form

$$T(x, y) = \sum_{j_1, j_2=-5}^{+5} \sum_{n=0}^5 C_{n, \ell_1 \ell_2} H_n(x, y) B(x, y) \quad (C.4)$$

with $0 < T(x, y) < 1$ for all x, y and $B(x, y) = \exp \left[\frac{1}{w_c^2} (x^2 + y^2) \right]$. Note, $T(x, y)$ is a real number while H_n and $C_{n, \ell_1 \ell_2}$ may be complex numbers. The Gaussian responses give

$$K(j_1 j_2 n) = \int \frac{dx dy}{(Nc)^2} T(x, y) H_n(x, y) W(j_1, j_2) / B(x, y),$$

and

$$W(j_1, j_2) = \exp [-2\pi f \underline{i}(x j_1 + y j_2)]. \quad (C.5)$$

We make the discrete analysis assumptions

$$x_\ell = c(\ell + \frac{1}{2}), \text{ and } y_{\bar{\ell}} = (c(\bar{\ell} + \frac{1}{2})). \quad (C.6)$$

Thus, the complete orthogonal function transformation sequence is given as

$$K(j_1, j_2, n) = \sum_{\ell, \bar{\ell}=-N/2}^{N/2-1} W(j_1, j_2) T(x_\ell, y_{\bar{\ell}}) H(x_\ell, y_{\bar{\ell}}) / [N^2 B(x_\ell, y_{\bar{\ell}})], \quad (C.7)$$

and

$$T(x_\ell, y_{\bar{\ell}}) = \sum_{j_1=-5}^5 \sum_{j_2=-5}^5 \sum_{n=0}^5 C_{n, j_1, j_2} H_n(x_\ell, y_{\bar{\ell}}) B(x_\ell, y_{\bar{\ell}}) / W(j_1, j_2). \quad (C.8)$$

Here we assume $H_n(x_\ell, y_{\bar{\ell}})$ are real functions and have

$$C_{n, -j_1, -j_2}^* = C_{n, j_1, j_2}.$$

The $H_n(x_\ell, y_{\bar{\ell}})$ are chosen to be such that

$$F \equiv \sum_{\ell, \bar{\ell}} H_n(x_\ell, y_{\bar{\ell}}) H_{\bar{n}}(x_\ell, y_{\bar{\ell}}) W(-j_1 + \bar{j}_1, -j_2 + \bar{j}_2) / N^2. \quad (C.9)$$

Thus, $F = 0$ is $j_1 \neq \bar{j}_1$ and $j_2 \neq \bar{j}_2$ for all n and \bar{n} . If $j_1 = \bar{j}_1$ and $j_2 = \bar{j}_2$, the F is 0 if $n \neq \bar{n}$. Also, F is zero if $(j_1) = j_a(n)$ and $(j_2) \neq j_b(n)$. Finally, if $(j_1) = j_a(n)$, $(j_2) = j_b(n)$, then we require $F = 1$. These values imply $11 \times 11 \times 6 \times 6$ constraints on the form of $H_n(x_\ell, y_{\bar{\ell}})$.

We can construct a set of H_n functions by differentiation by x and y of a single function, D , and impose the constraint that all functions are nonzero in a finite region of x and y and by the constraint that

$$\sum_{\ell, \bar{\ell}} D(x_\ell, y_{\bar{\ell}})^2 W(j, j_1) = 0 \quad (C.10)$$

for the allowed set j, j_1 , namely, $0 < |j| \leq 5$, and $0 < |j_1| \leq 5$.

It is easy to show that the desired H_n functions are a linear combination of those functions generated by differentiation of D . The basic problem is devising the appropriate D function. To succeed will require computer simulation and appropriate application of spline theory for two dimensional variable [7,8] and a decision of the reference electric field that should be used to characterize the particular laser source. One example of such a reference would be the Hermite TEM₀₀ mode that has been heavily truncated by the transverse dimensions in the laser source.

Appendix D. Program GEN

```
10 REM PROGRAM GEN TO CREATE FILE GLNK AND H(N,K,L)
15 CLOSE 5
16 CLOSE 4
20 OPEN "M0:",5,1
25 OPEN "CEN:",4,1
27 REW 5
40 DIM H1(1,81)
70 GOSUB 1110
74 N8=N-1
79 FOR L=0 TO N8
77 GOSUB 1420
78 GOSUB 3000
79 PRINT "L=",L,"TIME IS ",GETTIME$(0)
81 NEXT L
82 WF 5
83 REW 5
84 CLOSE 4
85 PRINT "FILE GENERATED"
86 CLOSE 5
87 STOP
1110 REM INITIAL BLOCK
1111 DIM H(7),C(9),S(9),U(7)
1112 C=.2E-1
1113 C3=C*2
1114 W1=.2E-1
1115 C2=2*C
1116 W2=10
1117 Z=1000
1118 A1=1.064
1119 N=1750
1120 D1=10*W1
1121 G5=C2*SQR(2)/W2
1122 F=D1/A1
1124 W3=(1/W2)^2
1125 P1=3.141592654
1128 W4=(P1*W1/A1)^2
1130 W5=W3+W4
1132 W6=2*W4
1134 F1=P1*2*F
1135 GOSUB 1400
1210 REM SET GENERATOR FUNCTIONS
1212 C5=COS(F1*C)
1214 S5=SIN(F1*C)
1216 H(0)=1
1218 V1=EXP(-C3*W5)
1220 V2=EXP(-C3*W6)
1222 V3=V1^2
1224 V4=V2^2
1226 H(1)=-G5/2
1228 C(1)=COS(F1*C/2)
1230 S(1)=-SIN(F1*C/2)
1232 C(0)=1
1234 S(0)=0
1236 V5=EXP(-C3*W5*.25)
1238 V6=1
1240 V7=EXP(-C3*W6*.25)
1242 V8=1
1250 RETURN
1310 REM HERMITE SET N=0 TO 7
1312 H(1)=H(1)+G5
1314 FOR N1=2 TO 7
1316 H(N1)=H(1)*H(N1-1)-2*(N1-1)*H(N1-2)
1318 NEXT N1
1320 REM COS AND SIN SET
1322 C8=C(1)*C5-S(1)*S5
1324 S(1)=C(1)*S5+S(1)*C5
```

```
1326 C(1)=C8
1328 FOR K=2 TO 9
1330 C(K)=C(K-1)*C(1)-S(K-1)*S(1)
1332 S(K)=S(K-1)*C(1)+C(K-1)*S(1)
1334 NEXT K
1338 V5=V5*V6
1340 V7=V7*V8
1342 V6=V6*V3
1344 V8=V8*V4
1350 RETURN
1400 REM U(N) SET
1402 U(0)=SQR(2*C*N*SQR(2/P1)/W2)
1403 FOR N1=1 TO 7
1404 U(N1)=U(N1-1)*SQR(1/(N1*2))
1405 NEXT N1
1406 RETURN
1420 REM GENERATE H1(J) FOR EACH L CASE
1422 GOSUB 1310
1424 H1(1,81)=V7
1426 FOR N1=0 TO 7
1427 M2=(-1)^N1
1428 H4=H(N1)*U(N1)*V5
1430 FOR K=0 TO 9
1432 J=N1+K*8+1
1434 IF M2<0 GOTO 1440
1436 H1(1,J)=H4*C(K)
1438 GOTO 1450
1440 H1(1,J)=H4*S(K)
1450 NEXT K
1455 NEXT N1
1460 RETURN
3000 REM OUTFILE
3010 MAT OUTFILE ON (5) H1
3012 MAT H1=(0)
3020 RETURN
20001 END
```

Appendix E. Program SELECT

```

10 REM PROGRAM SELECT R(N)
20 PRINT "THRESHOLD VALUE. T1=?"
30 INPUT T1
40 CLOSE 5
50 CLOSE 4
60 CLOSE 3
70 CLOSE 2
80 N3=1
90 OPEN "FILTER",4,0
100 OPEN "STA4",2,0
110 REW 2
120 OPEN "CEN:",3,1
130 PRINT ON (3)"PROGRAM SELECT",GETDATE$(0),"TIME IS",GETTIME$(0)
140 OPEN "M0:",5,0
150 DIM A(1750),E(1750),R(9),H1(1,81),U(7),K(5)
160 DIM A5(11,250),A4(11,10)
170 PRINT "READ IN DATA FROM FILTER YES=1,NO=0"
180 INPUT A5
190 IF A5=1 GOTO 1060
200 PRINT "BASE VALUES,R(6).R(7),R(8),R(9)=?"
210 INPUT R(6),R(7),R(8),R(9)
220 PRINT ON (3)"BASE VALUES R(6) TO R(9)=",R(6),R(7),R(8),R(9)
230 N=1750
240 N2=N-1
250 C=.2E-1
260 W2=10
270 P1=3.141592654
280 GOSUB 2100
290 FOR N1=1 TO 7
300 U(N1)=U(N1-1)/SQR(2*N1)
310 NEXT N1
320 FOR N1=0 TO 5
330 PRINT "K(";N1;")=?"
340 INPUT K(N1)
350 R(N1)=K(N1)*U(N1)/D7 '
360 PRINT ON (3)"K(N1),R(N1),N1=",K(N1),R(N1),N1
370 NEXT N1
380 N3=0
390 PRINT "S8,START, INCREMENT, NUMBER OF CASES, THRESHOLD, T1=?"
400 INPUT S8,S9,N9,T1
410 PRINT "READ IN H1(1,J) VALUES YES=1 NO=0"
420 INPUT NB
430 IF NB=0 GOTO 1660
440 REW 5
450 FOR L2=0 TO 174
460 FOR L1=1 TO 10
470 MAT INFILE ON (5) H1
480 GOSUB 1610
490 FOR N1=0 TO 5
500 N4=N1+1
510 K=N1+8*(N1+2)+1
520 A4(N4,L1)=H1(1,K)
530 NEXT N1
540 A4(11,L1)=H1(1,81)
550 NEXT L1
560 MAT OUTFILE ON (2) A4
570 NEXT L2
580 FF 5
590 REW 2
600 L3=-1
610 L=0
620 FOR L2=0 TO 174
630 MAT INFILE ON (2) A4
640 FOR L1=1 TO 10
650 L=L+1
660 L3=L3+1

```

```

670 FOR K=1 TO 11
680 A5(K,L)=A4(K,L1)
690 NEXT K
700 IF L<250 GOTO 730
710 MAT OUTFILE ON (5) A5
720 L=0
730 REM
740 NEXT L1
750 NEXT L2
760 CLOSE 2
770 PRINT ON (3)"FILE TRANSFER TIME IS ",GETTIME$(0)
780 WF 5
790 BF 5
800 IF N3>0 GOTO 1000
810 N3=1
820 GOSUB 1800
830 FOR L2=0 TO 6
840 MAT INFILE ON (5) A5
850 FOR L1=1 TO 250
860 L=L1-1+250*L2
870 A(L)=0
880 FOR K=0 TO 9
890 K1=K+1
900 A(L)=A(L)+A5(K1,L1)*R(K)
910 NEXT K
920 A(L)=A(L)/A5(11,L1)
930 GOSUB 1530
940 NEXT L1
950 NEXT L2
960 PRINT ON (3)"LARGEST A(";L3;")=",A8
970 PRINT "LARGEST A(";L3;")=",A8
980 PRINT "SMALLEST A(";L5;")=",A9
990 PRINT ON (3)"SMALLEST A(";L5;")=",A9
1000 S8=S8-S9
1010 FOR J8=1 TO N9
1020 S8=S8+S9
1030 GOSUB 1710
1040 NEXT J8
1050 GOTO 1210
1060 REM READ IN N, R(N),A(L) ETC.
1070 REW 4
1080 INFILE ON (4) N,U(6),U(7),R(6),R(7),R(8),R(9),D7
1090 PRINT ON (3)"N,U(6),U(7),R(6)-R(9),D7="
1100 PRINT ON (3) N,U(6),U(7),R(6),R(7),R(8),R(9),D7
1110 PRINT ON (3)"R(N1),U(N1),K(N1),N1"
1120 FOR N1=0 TO 5
1130 INFILE ON (4) R(N1),U(N1)
1140 PRINT ON (3) R(N1),U(N1),R(N1)*U(N1)*D7,N1
1150 NEXT N1
1160 N2=N-1
1170 FOR L=0 TO N2
1180 INFILE ON (4) A(L),E(L)
1190 NEXT L
1200 PRINT ON (3)" FILTER TRANSFER TIME IS ",GETTIME$(0)
1210 PRINT "NEW K(N)=1, NEW SCALE=0, NO=-1"
1220 INPUT A5
1230 IF A5=1 THEN 320
1240 IF A5=0 THEN 390
1260 REW 5
1270 GOSUB 1320
1280 CLOSE 5
1290 CLOSE 4
1300 PRINT "DONE","TIME IS ",GETTIME$(0)
1310 STOP
1320 REM PRINT TO FILTER DATA
1330 REW 4
1350 PRINT "SELECT S8 SCALE AND T1 THRESHOLD VALUES="
1360 INPUT S8,T1

```

```

1361 FOR I=0 TO 9
1362 R(I)=R(I)*S8
1363 NEKT I
1370 OUTFILE ON (4) N,U(6),U(7),R(6),R(7),R(8),R(9),D7
1375 CLOSE 3
1380 FOR N1=0 TO 5
1390 OUTFILE ON (4) R(N1),U(N1)
1400 NEKT N1
1410 N2=N-1
1420 A7=0
1430 FOR L=0 TO N2
1440 L4=N2-L
1450 A(L4)=S8*A(L4)
1460 A6=A(L4)
1470 GOSUB 1970
1480 NEKT L
1490 FOR L=0 TO N2
1500 OUTFILE ON (4) A(L),E(L)
1510 NEKT L
1520 RETURN
1530 REM SUBROUTINE FOR SIZE
1540 IF A8>A(L) GOTO 1570
1550 A8=A(L)
1560 L3=L
1570 IF A9<A(L) GOTO 1600
1580 A9=A(L)
1590 L5=L
1600 RETURN
1610 A4(7,L1)=H1(1,1)
1620 A4(8,L1)=H1(1,3)
1630 A4(9,L1)=H1(1,5)
1640 A4(10,L1)=H1(1,7)
1650 RETURN
1660 REW 5
1670 FF 5
1680 FF 5
1690 BF 5
1700 GOTO 800
1710 REM GENERATE E(L) GIVEN A(L), S8
1720 A2=0
1730 T2=1
1740 A7=0
1750 FOR L=0 TO N2
1760 L4=N2-L
1770 A6=S8*A(L4)
1780 GOSUB 1970
1790 A2=A2+A7+2
1800 NEKT L
1810 E1=A7/N
1820 E2=A2/N
1830 E3=SQR(E2-E1*E1)
1848 PRINT ON (3)"ERROR E1,E3. ",E1,E3,"SCALE=".S8
1850 PRINT ON (3)"TIME ",GETTIME$(0)
1860 PRINT ON (3)"THRESHOLD VALUE=",T1
1870 RETURN
1880 BF 5
1890 FF 5
1900 A8=1
1910 L3=-1
1920 L5=1750
1930 A9=1
1940 B8=0
1950 B9=0
1960 RETURN
1970 REM SUBROUTINE TO SET THE FILTER
1980 REM START AT L4=N2 WITH E(N2)=0
1990 IF L>0 GOTO 2030
2000 E(L4)=0
2010 A7=A7+A6

```

```
2020 RETURN
2030 A7=A7+A6
2040 IF A7>T1 GOTO 2070
2050 E(L4)=0
2060 RETURN
2070 E(L4)=1
2080 A7=A7-T2
2090 RETURN
2100 REM SUB
2110 D7=C4N*SQR(2*T1)*L2
2120 U(0)=SQR(2*D7)
2130 RETURN
2140 CLOSE 3
2150 OPEN "C:\N\$,3,1
2160 FOR L=0 TO N2
2170 PRINT ON (3) A(L),E(L).L;
2180 NEXT L
2190 CLOSE 3
2200 END
```

Appendix F. Program ORTHBAS

```
1 REM PROGRAM ORTHBAS
10 CLOSE 5
20 CLOSE 3
30 OPEN "M0:*.5.2"
40 OPEN "CEN:*.3.1"
50 REW 5
55 PRINT "X1,X2,X3=?"
56 INPUT X1,X2,X3
57 PRINT ON (3)"X1,X2,X3=",X1,X2,X3
60 PRINT ON (3)"TIME START ",GETTIME$(0)
70 DIM H(1,81)
71 DIM S(3325)
79 K=-1
80 FOR J=X1 TO X2
81 FOR J1=J TO X3
82 K=K+1
83 S(K)=0
84 NEXT J1
85 NEXT J
86 K1=K
87 PRINT "K1=",K1
88 STOP
90 FOR L=0 TO 1749
95 MAT INFILE ON (5) H
100 K=-1
110 FOR J=X1 TO X2
112 X=H(1,J)/H(1,81)
115 FOR J1=J TO X3
120 K=K+1
125 S(K)=S(K)+H(1,J1)*X
130 NEXT J1
135 NEXT J
136 PRINT "#L#",L,GETTIME$(0)
140 NEXT L
150 REW 5
160 CLOSE 5
165 OPEN "ORTHO",5,0
170 FOR K=0 TO K1
180 OUTFILE ON (5) S(K)/875
181 FOR I=1 UN (3)"S(";K,")=";S(K),
185 NEXT K
186 PRINT ON (3)"END OF DATA OUTPUT      ",GETTIME$(0)
200 STOP
210 END
```

Appendix G. Program CHECK

```
10 REM PROGRAM CHECK AND REDUCE OF S SET WITH ERROR CHECK
20 CLOSE 4
25 CLOSE 5
30 CLOSE 3
40 OPEN "M0:",5,2
50 OPEN "CEN:",3,1
60 OPEN "FILTER",4,0
65 REW 5
66 PRINT ON (3)"TIME START",GETTIME$(0)
67 PRINT "TIME START",GETTIME$(0)
68 PRINT ON (3)"PROGRAM CHECK DATE OF RUN IS ".GETDATE$(0)
70 REW 4
71 DIM H1(1,81),U(7),R(9),E(1750),A(1750),E1(1750)
72 INFILE ON (4) N,U(6),U(7),R(6),R(7),R(8),R(9),D7
73 PRINT ON (3)"N,U(6),U(7),R(6)-R(9),D7"
74 PRINT ON (3) N,U(6),U(7),R(6),R(7),R(8),R(9),D7
75 FOR N1=0 TO 5
76 INFILE ON (4) R(N1),U(N1)
77 PRINT ON (3)"R,U,K,WITH N1=",N1
78 PRINT ON (3) R(N1),U(N1).R(N1)*U(N1)*D7
80 NEXT N1
81 N2=N-1
85 FOR L=0 TO N2
90 INFILE ON (4) A(L),E(L)
95 NEXT L
100 REW 4
105 CLOSE 4
110 DIM S1(1,81),S(1,81),A2(1750)
120 FOR L=0 TO N2
125 MAT INFILE ON (5) H1
130 FOR J=1 TO 80
134 G8=E(L)-A(L)
135 S(1,J)=S(1,J)+H1(1,J)*G8
136 NEXT J
140 NEXT L
145 REW 5
146 PRINT ON (3)"TIME FOR S",GETTIME$(0)
147 PRINT "TIME FOR S",GETTIME$(0)
148 N8=N/2
150 FOR N1=0 TO 7
151 N3=(-1)^N1
152 FOR K=0 TO 9
153 J=K*8+N1+1
154 IF K>0 GOTO 156
155 GOSUB 1000
156 S(1,J)=S(1,J)/N8
158 NEXT K
160 NEXT N1
165 PRINT "SET FOR REDUCE IS 1 FOR NO AND 0 FOR YES"
166 INPUT R
168 IF R=1 GOTO 200
169 GOSUB 2000
200 FOR L=0 TO N2
201 E1(L)=0
205 MAT INFILE ON (5) H1
208 FOR J=1 TO 80
210 E1(L)=E1(L)+H1(1,J)*S(1,J)
212 NEXT J
213 E1(L)=E1(L)/H1(1,81)
215 NEXT L
216 REW 5
270 PRINT ON (3)"TIME FOR S1",GETTIME$(0)
271 PRINT "TIME FOR S1",GETTIME$(0)
300 MAT S1=(0)
310 FOR L=0 TO N2
315 MAT INFILE ON (5) H1
320 FOR J=1 TO 80
```

```

325 S1(1,J)=H1(1,J)*E1(L)+S1(1,J)
326 NEXT J
330 NEXT L
340 REW 5
350 FOR N1=0 TO 7
355 N3=(-1)^N1
360 FOR K=0 TO 9
365 J=1+K*8+N1
370 IF K>0 GOTO 375
371 GOSUB 1100
375 S1(1,J)=S1(1,J)/N3-S(1,J)
380 NEXT K
385 NEXT N1
389 PRINT ON (3)"S VALUES",GETTIME$(0)
390 MAT PRINT ON (3) S
391 PRINT "S AND S1 COMPLETED",GETTIME$(0)
394 PRINT ON (3)"DIFFERENCE IN S VALUES DUE TO TRANSFORM"
395 MAT PRINT ON (3) S1
396 OPEN "TRUE",4,0
397 A3=0
398 REW 4
399 PRINT ON (3)"L,A,E,E1,E-E1 LIST "
400 REM
402 FOR L=0 TO N2
403 OUTFILE ON (4) A(L),E(L),E1(L)
404 PRINT ON (3) L,A(L),E(L),E1(L),E(L)-E1(L)
405 NEXT L
500 PRINT "DONE",GETTIME$(0)
510 REW 4
520 REW 5
530 CLOSE 3
540 CLOSE 4
545 CLOSE 5
550 STOP
1000 REM
1001 IF N3<0 GOTO 1010
1005 S(1,J)=S(1,J)/2
1006 GOTO 1015
1010 S(1,J)=0
1015 RETURN
1100 REM
1101 IF N3<0 GOTO 1110
1105 S1(1,J)=S1(1,J)/2
1106 GOTO 1115
1110 S1(1,J)=0
1115 RETURN
2000 REM REDUCE OF AREA IN COMPUTATION
2001 FOR N1=0 TO 7
2002 FOR K=0 TO 9
2003 J=8*K+N1-1
2004 IF K<2 OR K>7 GOTO 2006
2005 GOTO 2007
2006 S(1,J)=0
2007 NEXT K
2009 J=8*(N1+2)+N1-1
2010 S(1,J)=0
2011 NEXT N1
2015 RETURN
6000 END

```

Appendix H. Program CONVERGE

```

10 REM PROGRAM CONVERGE
15 DATA 2,1,2,1,3,0,3,1,3,1,3,1
16 CLOSE 2
17 OPEN "CEN:",2,0
20 GOSUB 1100
25 PRINT "START TIME ",GETTIME$(0)
30 PRINT "READ INITIAL FILTER YES=1.NO=0. ITERATION LEVEL L9="
35 INPUT I8,I9
40 IF I8=0 GOTO 90
45 GOSUB 1000
46 GOSUB 1250
50 GOSUB 1500
54 PRINT ON (3)"S WITH R",GETTIME$(0)
55 GOSUB 1800
60 GOSUB 1700
64 PRINT ON (3)"S WITHOUT R",GETTIME$(0)
65 GOSUB 1800
70 GOSUB 1600
75 GOSUB 2000
80 GOSUB 1400
85 GOTO 95
90 GOSUB 1250
91 GOSUB 1300
92 I9=I9+1
95 GOSUB 1900
485 I9=I9-1
486 PRINT ON (3)"ITERATION VALUE IS",I9,GETTIME$(0)
487 PRINT "ITERATION VALUE IS ",I9,GETTIME$(0)
490 IF I9>0 GOTO 50
495 GOSUB 1280
500 GOSUB 1200
510 PRINT "TIME STOP",GETTIME$(0)
515 STOP
1000 REM SUBROUTINE TO READ FILTER AND PRINT STATUS
1001 PRINT "1000"
1010 CLOSE 4
1020 OPEN "FILTER ",4,0
1025 REW 4
1030 INFILE ON (4) N,U(6),U(7),R(6),R(7),R(8),R(9),D?
1035 PRINT ON (3)"N,U(6),U(7),R(6)-R(9),D?"
1040 PRINT ON (3) N,U(6),U(7),R(6),R(7),R(8),R(9),D?
1045 FOR N1=0 TO 5
1050 INFILE ON (4) R(N1),U(N1)
1055 PRINT ON (3)"R,U,K,WITH N1=",N1
1060 PRINT ON (3) R(N1),U(N1),R(N1)*D?/U(N1)
1065 NEXT N1
1070 N2=N-1
1075 FOR L=0 TO N2
1080 INFILE ON (4) A(L),E(L)
1085 NEXT L
1090 REW 4
1095 CLOSE 4
1099 RETURN
1100 REM SUBROUTINE TO SET INTITAL CONDITIONS
1101 PRINT "1100"
1102 N4=1750
1105 DIM H(1,81),S(1,81),E(N4),A(N4),E1(N4)
1106 DIM U(7),R(9)
1110 CLOSE 5
1115 CLOSE 3
1120 OPEN "M0:",5,2
1125 OPEN "CEN:",3,1
1130 REW 5
1135 PRINT ON (3)"TIME START",GETTIME$(0)
1140 PRINT ON (3)"PROGRAM CONVERGE, DATE IS ",GETDATE$(0)
1145 RETURN

```

```

1200 REM SUBROUTINE TO CLOSE PROGRAM
1201 PRINT "1200"
1205 REW 5
1210 CLOSE 5
1211 PRINT ON (3)"A(L),E(L),E1(L),L"
1212 FOR L=0 TO N2
1213 PRINT ON (3) A(L),E(L),E1(L),L
1214 NEXT L
1215 PRINT ON (3)"TIME STOP",GETTIME$(0)
1220 CLOSE 3
1222 CLOSE 2
1225 RETURN
1250 REM SUBROUTINE TO OPEN TRUE
1251 PRINT "1250"
1255 CLOSE 4
1265 OPEN "TRUE ",4,0
1266 REW 4
1270 RETURN
1280 REM SUBROUTINE TO CLOSE TRUE
1281 PRINT "1280"
1285 REW 4
1290 CLOSE 4
1295 RETURN
1300 REM SUBROUTINE TO READ TRUE
1301 PRINT "1300"
1305 REW 4
1310 FOR N1=0 TO 9
1315 INFILE ON (4) R(N1)
1320 NEXT N1
1325 FOR J=1 TO 81
1330 INFILE ON (4) S(1,J)
1335 NEXT J
1340 INFILE ON (4) S2,S3,N
1341 N2=N-1
1345 FOR L=0 TO N2
1350 INFILE ON (4) A(L),E(L),E1(L)
1355 NEXT L
1360 RETURN
1400 REM SUBROUTINE TO PRINT TO TRUE
1401 PRINT "1400"
1405 REW 4
1410 FOR N1=0 TO 9
1415 OUTFILE ON (4) R(N1)
1420 NEXT N1
1425 FOR J=1 TO 81
1430 OUTFILE ON (4) S(1,J)
1435 NEXT J
1440 OUTFILE ON (4) S2,S3,N
1442 PRINT ON (3)"N4,N,N2=".N4,N,N2
1445 FOR L=0 TO N2
1450 OUTFILE ON (4) A(L),E(L),E1(L)
1455 NEXT L
1460 RETURN
1500 REM SUBROUTINE TO GENERATE S,S2,S3 GIVEN E(L) A(L)
1501 PRINT "1500"
1505 S3=0
1510 S2=0
1511 REW 5
1515 MAT S=(0)
1520 FOR L=0 TO N2
1525 MAT INFILE ON (5) H
1530 FOR J=1 TO 80
1534 A7=E(L)
1535 S(1,J)=S(1,J)+H(1,J)*A7
1540 NEXT J
1545 S3=S3+(E(L)+2)*H(1,81)
1550 S2=S2+(A(L)+2)*H(1,81)

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```

1555 NEXT L
1560 REW 5
1561 N8=N/2
1562 FOR N1=0 TO 7
1563 N3=(-1)^N1
1564 FOR K=0 TO 9
1565 J=8*K+N1+1
1566 IF K>0 GOTO 1572
1567 IF N3<0 GOTO 1570
1568 S(1,J)=S(1,J)/2
1569 GOTO 1572
1570 S(1,J)=0
1572 S(1,J)=S(1,J)/N8
1573 NEXT K
1574 NEXT N1
1575 S2=S2/N8
1576 S3=S3/N8
1577 RETURN
1600 REM SUBROUTINE TO GENERATE E1 GIVEN S
1601 PRINT "1600"
1605 FOR L=0 TO N2
1610 E1(L)=0
1615 MAT INFILE ON (5) H
1620 FOR J=1 TO 80
1625 E1(L)=E1(L)+H(1,J)*S(1,J)
1630 NEXT J
1635 E1(L)=E1(L)/H(1,B1)
1640 NEXT L
1645 REW 5
1650 RETURN
1700 REM SUBROUTINE TO REMOVE R(N1) FROM S
1701 PRINT "1700"
1705 S(1,1)=S(1,1)-R(6)
1706 S(1,3)=S(1,3)-R(7)
1707 S(1,5)=S(1,5)-R(8)
1709 S(1,7)=S(1,7)-R(9)
1710 S(1,17)=S(1,17)-R(0)
1712 S(1,26)=S(1,26)-R(1)
1714 S(1,35)=S(1,35)-R(2)
1716 S(1,44)=S(1,44)-R(3)
1718 S(1,53)=S(1,53)-R(4)
1720 S(1,62)=S(1,62)-R(5)
1725 FOR K=0 TO 9
1726 FOR N1=0 TO 7
1727 J=8*K+N1+1
1728 IF K>1 AND K<8 GOTO 1735
1729 S(1,J)=0
1735 IF N1<6 GOTO 1740
1736 S(1,J)=0
1740 NEXT N1
1745 NEXT K
1750 RETURN
1800 REM SUBROUTINE TO PRINT S
1801 PRINT "1800"
1810 MAT PRINT ON (3) S
1815 S5=0
1820 FOR J=1 TO 80
1825 S5=S5+S(1,J)^2
1830 NEXT J
1831 FOR J=1 TO 8
1832 S5=S5+S(1,J)^2
1833 NEXT J
1840 PRINT ON (3)"S^2,S5=",S5
1850 RETURN
1900 REM PRINT R,S2,S3,S4
1901 PRINT "1900"
1905 PRINT ON (3)"R SET",GETTIME$(0)
1908 S4=0
1910 FOR N1=0 TO 9

```

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1915 PRINT ON (3)"R(";N1;")=",R(N1)
1920 S4=S4+R(N1)^2
1921 IF N1<6 GOTO 1925
1922 S4=S4+R(N1)^2
1925 NEXT N1
1930 PRINT ON (3)"E^2 SUM S2=";S2,"A^2 SUM S3=";S3
1935 PRINT ON (3)"R^2 SUM S4=";S4
1940 RETURN
2000 REM GENERATE E(L) GIVEN A(L),E1(L),R8
2001 PRINT "2000"
2004 READ L6,R7
2005 R8=L6-1
2006 PRINT ON (3)"QUANTIZED L6=",L6,"RELAXATION R7=",R7
2010 L5=1/R8
2015 J5=1/L6
2018 L8=1
2020 A7=0
2021 B7=0
2025 FOR L=0 TO N2
2030 L4=N2-L
2035 A6=A(L4)-E1(L4)*R7
2040 A7=A7+A6
2045 B1=L8
2050 B2=L7
2055 FOR J=1 TO L6
2060 B1=B1-J5
2065 B2=B2-L5
2070 IF A7>B1 GOTO 2080
2075 NEXT J
2080 E(L4)=B2
2081 B7=B7+B2
2085 A7=A7-B2
2095 NEXT L
2098 PRINT ON (3)"SUM OF E(L) IS ",B7,"NORMAL B7/N =",B7/N
2099 PRINT ON (3)"REMAINDER A7=",A7
2100 RETURN
2109 END

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10. SUPPLEMENTARY NOTES <p><input type="checkbox"/> Document describes a computer program; SF-185, FIPS Software Summary, is attached.</p> <p>11. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)</p> <p>As a first step in the development of a beam-profile measuring instrument for laser sources that is capable of determining the distribution of low-order (less than 25) Hermitian modes in a series of laser pulses, I designed and evaluated the three key parts of such an instrument. First, there is the telescope system which allows the incident laser beam to be phase, beamwidth, and beam center matched to the optical spatial filter. Second, there is a brief error analysis of the structure of the mismatch function between the beam out of the telescope and that expected by the filter. Finally, there is the detailed analysis and design of the computer-generated spatial filter that will cause the incident-laser beam to be cross correlated with the low-order Hermite modes and will create an array of light spots in the detector (Fourier transform) plane each of which can be uniquely related to a particular Hermite mode of the original laser pulse.</p> <p>The principal conclusion is that the Hermite mode analysis can be done with better than 99 percent separation between modes, provided the phase between modes is uncorrelated from pulse to pulse when the filter has been fabricated with a two-level, gray-scale structure which samples the profile with either 0 percent, or 100 percent transmission.</p>			
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